PHYSICS 410

TRIDIAGONAL STRUCTURE OF FDA FOR DIFFUSION EQUATION

- $O(\Delta t.\Delta x^2)$ implicit scheme for diffusion equation can be written as

$$\left(-\Delta x^{-2}\right) u_{j+1}^{n+1} + (\Delta t^{-1} + 2\Delta x^{-2}) u_j^{n+1} + \left(-\Delta x^{-2}\right) u_{j-1}^{n+1} = \left(\Delta t^{-1}\right) u_j^n, \quad j = 2, 3, \ldots J-1$$

$$u_1^{n+1} = u_J^{n+1} = 0$$

or

$$c_j^+ u_{j+1}^{n+1} + c_j^0 u_j^{n+1} + c_j^- u_{j-1}^{n+1} = f_j, \quad j = 2, 3, \ldots J-1$$

where c_j^+, c_j^0, c_j^- and f_j are coefficients and/or quantities that are known at time step t^n

- In matrix form we have, schematically

$$\begin{bmatrix} 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \star & \star & \star & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \star & \star & \star & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \star & \star & \star & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & c_j^- & c_j^0 & c_j^+ & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \star & \star & \star & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \star & \star & \star & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \star & \star & \star \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1^{n+1} \\ u_2^{n+1} \\ \star \\ u_{j-1}^{n+1} \\ u_j^{n+1} \\ u_{j+1}^{n+1} \\ \star \\ u_{J-1}^{n+1} \\ u_J^{n+1} \end{bmatrix} = \begin{bmatrix} 0 \\ f_2 \\ \star \\ \star \\ f_j \\ \star \\ \star \\ f_{J-1} \\ 0 \end{bmatrix}$$

where $\cdot$ and $\star$ denote a zero and non-zero entry, respectively.

- The matrix is clearly tridiagonal

- Also note how the boundary conditions are incorporated