# Source: Maple V Learning Guide (ver. 2, Sep 24, 2013)

## 2. Mathematics with Maple: The Basics

### 2.1 Introduction

**NOTE:** When entering commands using "Maple Notation" vs "2-D Math Notation" (see Tools->Options->Display->Input Display), and especially when programming in Maple (writing scripts/procedures), **you should be sure to end every statement with a semi-colon, or colon: the colon suppresses output.**

```
> 1 + 2;
```

$$3$$

```
> 1 + 3/2;
```

$$\frac{5}{2}$$

```
> 2*(3 + 1/3)/(5/3 - 4/5);
```

$$\frac{100}{13}$$

```
> 2.8754/2;
```

$$1.437700000$$

```
> 1+1/2;
```

$$\frac{3}{2}$$

### 2.2 Numerical Computations

#### Integer computations

```
> 1 + 2;
```

$$3$$

```
> 75 - 3;
```

$$72$$

```
> 5*3;
```

$$15$$

```
> 120/2;
```

$$60$$

```
> 100!;
```

$$933262154439441526816992388562667004907159682643816214685929638952175999932299156089414639761565182862536979208272237582511852109168640000000000000000000000$$

Note the use of % in the following, which is short-hand for "last result". Similarly, %% and %%% can be used to retrieve the second-to-last and third-to-last results respectively.

```
> length(%);
```

$$158$$

**Commands for Working With Integers**

```
> ifactor(60);
```

$$(2)^2\,(3)\,(5)$$

```
> igcd(123, 45);
```

$$3$$

```
> iquo(25, 3);
```

$$8$$

```
> isprime(18002676583);
```

$$true$$

**Exact Arithmetic - Rationals, Irrationals, and Constants**

```
> 1/2 + 1/3;
```

$$\frac{5}{6}$$

```
> Pi;
```

$$\pi$$

```
> evalf(Pi, 100);
```

$$3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706 8$$

```
> 1/3;
```

$$\frac{1}{3}$$

```
> evalf(%);
```

$$0.3333333333$$

```
> 3/2*5;
```

$$\frac{15}{2}$$

```
> 1.5*5;
```

$$7.5$$

```
> sqrt(2);
```

$$\sqrt{2}$$

```
> sqrt(3)^2;
```

$$3$$

```
> Pi;
```

$$\pi$$

```
> sin(Pi);
```

$$0$$

```
> exp(1);
```

$$e$$

```
> ln(exp(5));
```

$$5$$

```
> 1/3 + 1/4 + 1/5.3;
```

$$0.7720125786$$

```
> sin(0.2);
```

$$0.1986693308$$

```
> Digits := 20;
```

$$Digits := 20$$

```
> sin(0.2);
```

$$0.19866933079506121546$$

Restore default number of digits for floating point expressions (10)

```
> Digits := 10;
```

$$Digits := 10$$

**Arithmetic with Special Numbers**

**Note: I = sqrt(-1)**

```
> (2 + 5*I) + (1 - I);
```

$$3 + 4\,I$$

```
> (1 + I)/(3 - 2*I);
```

$$\frac{1}{13} + \frac{5}{13}\cdot I$$

```
> convert(247, binary);
```

$$11110111$$

```
> convert(1023, hex);
```

$$3FF$$

```
> 27 mod 4;
```

$$3$$

**Mathematical Functions**

```
> sin(Pi/4);
```

$$\frac{1}{2}\sqrt{2}$$

```
> ln(1);
```

$$0$$

```
> ln(Pi);
```

$$\ln(\pi)$$

## 2.3 Basic Symbolic Computations

```
> (1 + x)^2;
```

$$(1+x)^2$$

```
> (1 + x) + (3 - 2*x);
```

$$4-x$$

```
> expand((1 + x)^2);
```

$$1+2x+x^2$$

```
> factor(%);
```

$$(1+x)^2$$

Symbolic differentiation and integration

Maple has separate 'inert' (upper-case) and 'active' (lower-case) commands for differentiation/integration; the 'inert' forms return the usual mathematical representation of the operations, unevaluated

```
> Diff(sin(x), x);
```

$$\frac{d}{dx}\sin(x)$$

```
> value(%);
```

$$\cos(x)$$

```
> diff(sin(x), x);
```

$$\cos(x)$$

```
> Int(cos(x)^2, x);
```

$$\int \cos(x)^2\,dx$$

```
> value(%);
```

$$\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

```
> int(cos(x)^2, x);
```

$$\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

```
> Sum(n^2, n);
```

$$\sum_n n^2$$

```
> value(%);
```

$$\frac{1}{3} n^3 - \frac{1}{2} n^2 + \frac{1}{6} n$$

```
> sum(n^2, n);
```

$$\frac{1}{3} n^3 - \frac{1}{2} n^2 + \frac{1}{6} n$$

```
> rem(x^3 + x + 1, x^2 + x + 1, x);
```

$$2 + x$$

```
> series(sin(x), x = 0, 10);
```

$$x - \frac{1}{6} x^3 + \frac{1}{120} x^5 - \frac{1}{5040} x^7 + \frac{1}{362880} x^9 + O(x^{11})$$

## 2.4 Assigning Expressions to Names

### Syntax for Naming an Object (a.ka. Variable)

**name := expression;**

```
> var := x;
```

$$var := x$$

```
> term := x*y;
```

$$term := x\,y$$

```
> eqns := x = y + 2;
```

$$eqns := x = y + 2$$

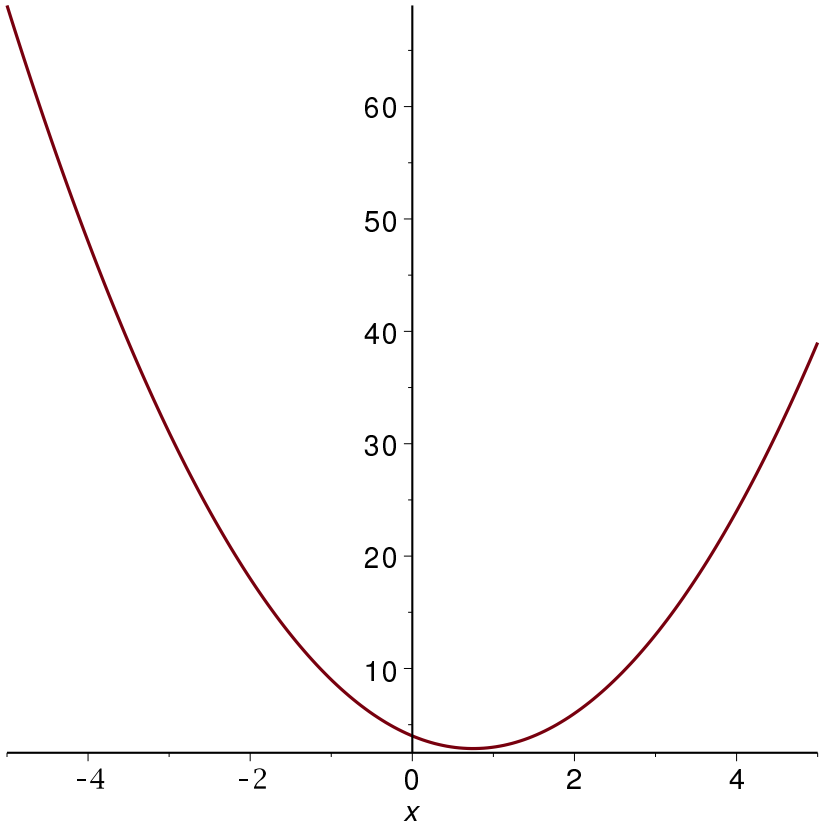
**BEWARE OF USING = WHEN YOU MEAN := !!!**

### Maple Arrow Notation for Defining functions

```
> f := x -> 2*x^2 - 3*x + 4;
```

$$f := x \rightarrow 2 x^2 - 3 x + 4$$

```
> plot(f(x), x = -5 .. 5);
```




**More on the Assignment Operator**

```
> f := x -> x^2;
```

$$f := x \to x^2$$

```
> f(5);
```

$$25$$

```
> f(y+1);
```

$$(y+1)^2$$

**Predefined and Reserved Names**

```
> Pi := 3.14;
Error, attempting to assign to `Pi` which is protected
```

```
> set := {1, 2, 3};
```

Error, attempting to assign to `set` which is protected

## 2.5 Basic Types of Maple Objects

### Expression sequences

```
> 1, 2, 3, 4;
```

$$1, 2, 3, 4$$

```
> x, y, z, w;
```

$$x, y, z, w$$

```
> 10, x, sin(y), (w + z)^3, 3.0;
```

$$10, x, \sin(y), (w+z)^3, 3.0$$

### Simultaneous assignment of sequence of names (variables)

```
> f, g, h := 3, 6, 1;
```

$$f, g, h := 3, 6, 1$$

```
> f ;
```

$$3$$

```
> h;
```

$$1$$

### Lists

```
> data_list := [1, 2, 3, 4, 5];
```

$$data_list := [1, 2, 3, 4, 5]$$

```
> polynomials := [x^2 + 3, x^2 + 3*x - 1, 2*x];
```

$$polynomials := [x^2+3, x^2+3\,x-1, 2\,x]$$

```
> participants := [Kathy, Frank, Rene, Niklaus, Liz];
```

$$participants := [Kathy, Frank, Rene, Niklaus, Liz]$$

Order and repeats of elements in lists is preserved

```
> [a, b, c], [b, c ,a], [a, a, b, c, a];
```

$$[a, b, c], [b, c, a], [a, a, b, c, a]$$

```
> letters := [a, b, c];
```

$$letters := [a, b, c]$$

Extraction of list element using indexing (subscript) operator [ ]

```
> letters[2];
```

$$b$$

```
> letters[0];
```

Error, invalid subscript selector

```
> letters[4];
```

Error, invalid subscript selector

Length of list

```
> nops(letters);
```

$$3$$

**Sets**

```
> data_set := {1, -1, 0, 10, 2};
```

$$data_set := \{-1, 0, 1, 2, 10\}$$

```
> unknowns := {x, y, z};
```

$$unknowns := \{x, y, z\}$$

Order of elements in sets is arbitrary, no repeated elements

```
> {a, b, c}, {c, b, a}, {a, a, b, c, a};
```

$$\{a, b, c\}, \{a, b, c\}, \{a, b, c\}$$

```
> {1, 2, 2.0};
```

$$\{1, 2, 2.0\}$$

```
> {a, b, c} union {c, d, e};
```

$$\{a, b, c, d, e\}$$

```
> {1, 2, 3, a, b, c} intersect {0, 1, y, a};
```

$$\{1, a\}$$

```
> nops(%);
```

$$2$$

**More Operations on Sets and Lists**

```
> participants := [Kate, Tom, Steve];
```

$$participants := [Kate, Tom, Steve]$$

```
> member(Tom, participants);
```

$$true$$

```
> data_set := {5, 6, 3, 7};
```

$$data_set := \{3, 5, 6, 7\}$$

```
> member(2, data_set);
```

$$false$$

```
> participants[2];
```

$$Tom$$

```
> empty_set := {};
```

$$empty_set := \{\}$$

```
> empty_list := [];
```

$$empty_list := [\,]$$

```
> nops(empty_list);
```

$$0$$

```
> old_set := {2, 3, 4} union {};
```

$$old_set := \{2, 3, 4\}$$

```
> new_set := old_set union {2, 5};
```

$$new_set := \{2, 3, 4, 5\}$$

```
> third_set := old_set minus {2, 5};
```

$$third_set := \{3, 4\}$$

**Arrays (regularly structured collections of values/expressions)**

**1-dimensional arrays**

```
> squares := array(1..3);
```

$$squares := array(1..3, [\,])$$

Individual elements selected/specified using indexing operator [ ] as with lists, but lists and 1-dimensional arrays are distinct Maple objects

```
> squares[1] := 1; squares[2] := 2^2; squares[3] := 3^2;
```

$$squares_1 := 1$$

$$squares_2 := 4$$

$$squares_3 := 9$$

```
> cubes := array(1..3, [1, 8, 27]);
```

$$cubes := \begin{bmatrix} 1 & 8 & 27 \end{bmatrix}$$

Selection of single array element using [ ]

```
> squares[2];
```

$$4$$

Must "force evaluation" of array to display full set of values ...

```
> squares;
```

$$squares$$

... doesn't work, but ...

```
> print(squares);
```

$$\begin{bmatrix} 1 & 4 & 9 \end{bmatrix}$$

does as does ...

```
> evalm(squares);
```

$$\begin{bmatrix} 1 & 4 & 9 \end{bmatrix}$$

**2-dimensional arrays**

```
> pwrs := array(1..3, 1..3);
```

$$pwrs := array(1..3, 1..3, [\,])$$

Individual element definition / display uses "double index"

```
> pwrs[1,1] := 1; pwrs[1,2] := 1; pwrs[1,3] := 1;
```

$$pwrs_{1,1} := 1$$

$$pwrs_{1,2} := 1$$

$$pwrs_{1,3} := 1$$

```
> pwrs[2,1] := 2: pwrs[2,2] := 4: pwrs[2,3] :=8:
```

```
> pwrs[3,1] := 3: pwrs[3,2] := 9: pwrs[3,3] := 27:
```

Again, must explicitly force evaluation to display values

```
> print(pwrs);
```

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{bmatrix}$$

```
> pwrs[2,3];
```

$$8$$

```
> pwrs2 := array(1..3, 1..3, [[1, 1, 1], [2, 4, 8], [3, 9, 27]] );
```

$$pwrs2 := \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{bmatrix}$$

**3-dimensional arrays**

```
> array3 := array(1..2, 1..2, 1..2,
> [ [ [1, 2], [3, 4] ], [ [5,6], [7, 8] ] ] );
```

$$array3 := ARRAY([1..2, 1..2, 1..2], [(1, 1, 1) = 1, (1, 1, 2) = 2, (1, 2, 1) = 3, (1, 2, 2) = 4, (2, 1, 1) = 5, (2, 1, 2) = 6, (2, 2, 1) = 7, (2, 2, 2) = 8])$$

**The subs Command (substitutes sub-expressions with other expressions)**

```
> expr := z^2 + 3;
```

$$expr := z^2 + 3$$

```
> subs( {z = x + y}, expr );
```

$$(x + y)^2 + 3$$

Direct application of subs to an array object doesn't produce desired effect ...

```
> subs( {2 = 9}, pwrs );
```

$$pwrs$$

... so, again, force explicit evaluation of array elements

```
> pwrs3 := subs( {2 = 9}, evalm(pwrs) );
```

$$pwrs3 := \begin{bmatrix} 1 & 1 & 1 \\ 9 & 4 & 8 \\ 3 & 9 & 27 \end{bmatrix}$$

```
> evalm(pwrs3);
```

$$\begin{bmatrix} 1 & 1 & 1 \\ 9 & 4 & 8 \\ 3 & 9 & 27 \end{bmatrix}$$

See also the algsubs command (illustrated at end of worksheet), which is considerably more powerful and flexible. Also note that in the above expressions the braces { } around the substitutions to be performed are not necessary (but are if there is more than one substitution to be made)

**Strings (delimited with double quotes ", NOT single quotes ')**

```
> "This is a string.";
```

"This is a string."

```
> "my age" := 32;
Error, invalid left hand side of assignment
```

```
> mystr := "I ate the whole thing.";
```

*mystr* := "I ate the whole thing."

Substring extraction

```
> mystr[3..5];
```

"ate"

Negative range values index string from right, rather than left, end

```
> mystr[11..-2];
```

"whole thing"

```
> newstr := cat("I can't believe ", mystr);
```

*newstr* := "I can't believe I ate the whole thing."

```
> length(newstr);
```

## 2.6 Expression Manipulation

### The simplify Command

```
> expr := cos(x)^5 + sin(x)^4 + 2*cos(x)^2 -
>   2*sin(x)^2 - cos(2*x);
```

$$expr := \cos(x)^5 + \sin(x)^4 + 2\cos(x)^2 - 2\sin(x)^2 - \cos(2x)$$

```
> simplify(expr);
```

$$\cos(x)^4 (\cos(x) + 1)$$

```
> simplify(sin(x)^2 + ln(2*y) + cos(x)^2);
```

$$1 + \ln(2) + \ln(y)$$

```
> simplify(sin(x)^2 + ln(2*y) + cos(x)^2, 'trig');
```

$$1 + \ln(2y)$$

```
> simplify(sin(x)^2 + ln(2*y) + cos(x)^2, 'ln');
```

$$\sin(x)^2 + \ln(2) + \ln(y) + \cos(x)^2$$

### The factor Command

```
> big_poly := x^5 - x^4 - 7*x^3 + x^2 + 6*x;
```

$$big_poly := x^5 - x^4 - 7x^3 + x^2 + 6x$$

```
> factor(big_poly);
```

$$x(x-1)(x-3)(2+x)(1+x)$$

```
> rat_expr := (x^3 - y^3)/(x^4 - y^4);
```

$$rat_expr := \frac{x^3 - y^3}{x^4 - y^4}$$

```
> factor(rat_expr);
```

$$\frac{x^2 + xy + y^2}{(x+y)(x^2 + y^2)}$$

### The expand Command

```
> expand((x + 1)*(x + 2));
```

$$x^2 + 3x + 2$$

```
> expand(sin(x + y));
```

$$\sin(x)\cos(y)+\cos(x)\sin(y)$$

```
> expand(exp(a + ln(b)));
```

$$e^{a} b$$

**The convert Command**

```
> convert(cos(x), exp);
```

$$\frac{1}{2} e^{Ix} + \frac{1}{2} e^{-Ix}$$

```
> convert(1/2*exp(x) + 1/2*exp(-x), trig);
```

$$\cosh(x)$$

```
> A := array(1..2, 1..2, [ [a, b], [c, d] ]);
```

$$A := \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

```
> convert(A, 'listlist');
```

$$[[a, b], [c, d]]$$

```
> convert(A, 'set');
```

$$\{a, b, c, d\}$$

```
> convert(%, 'list');
```

$$[a, b, c, d]$$

**The combine Command**

Roughly speaking, combine can be viewed as an inverse of expand

```
> combine(exp(x)^2*exp(y), exp);
```

$$e^{2x+y}$$

```
> combine((x^a)^2, power);
```

$$x^{2a}$$

**The lhs and rhs Commands**

```
> eqn1 := x + y = z + 3;
```

$$eqn1 := x + y = z + 3$$

```
> lhs(eqn1);
```

$$x + y$$

```
> rhs(eqn1);
```

$$z + 3$$

**The numer and denom Commands**

```
> numer(3/4);
```

$$3$$

```
> denom(1/(1 + x));
```

$$1 + x$$

**Substitutions using subs and algsubs**

```
> expr := a^3*b^2;
```

$$expr := a^3 b^2$$

The following will make the substitution only when the expression literally contains the sub-expression a*b

```
> subs(a*b=5, expr);
```

$$a^3 b^2$$

Get the desired effect using algsubs

```
> algsubs(a*b=5, expr);
```

$$25 a$$