

Michael Pürrer,¹ Sascha Husa,^{2,3} and Peter C. Aichelburg¹

¹ *Institut für Theoretische Physik, Universität Wien, 1090 Wien, Austria*

² *Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, 14476 Golm, Germany*

³ *Departament de Física, Universitat de les Illes Balears,
Ctra de Valldemossa km 7,5, 07071 Palma de Mallorca, Spain*

(Dated: April 28, 2005)

We discuss critical gravitational collapse on the threshold of apparent horizon formation as a model both for the discussion of global aspects of critical collapse and for numerical studies in a compactified context. For our matter model we choose a self-gravitating massless scalar field in spherical symmetry, which has been studied extensively in the critical collapse literature. Our evolution system is based on Bondi coordinates, the mass function is used as an evolution variable to ensure regularity at null infinity. We compute radiation quantities like the Bondi mass and news function and find that they reflect the discretely self-similar (DSS) behavior. Surprisingly, the period of radiation at null infinity is related to the formal result for the leading quasi-normal mode of a black hole with rapidly decreasing mass. Furthermore, our investigations shed some light on global versus local issues in critical collapse, and the validity and usefulness of the concept of null infinity when predicting detector signals.

PACS numbers: 04.25.Dm, 04.20.Ha, 04.20.Dw, 04.30.-w

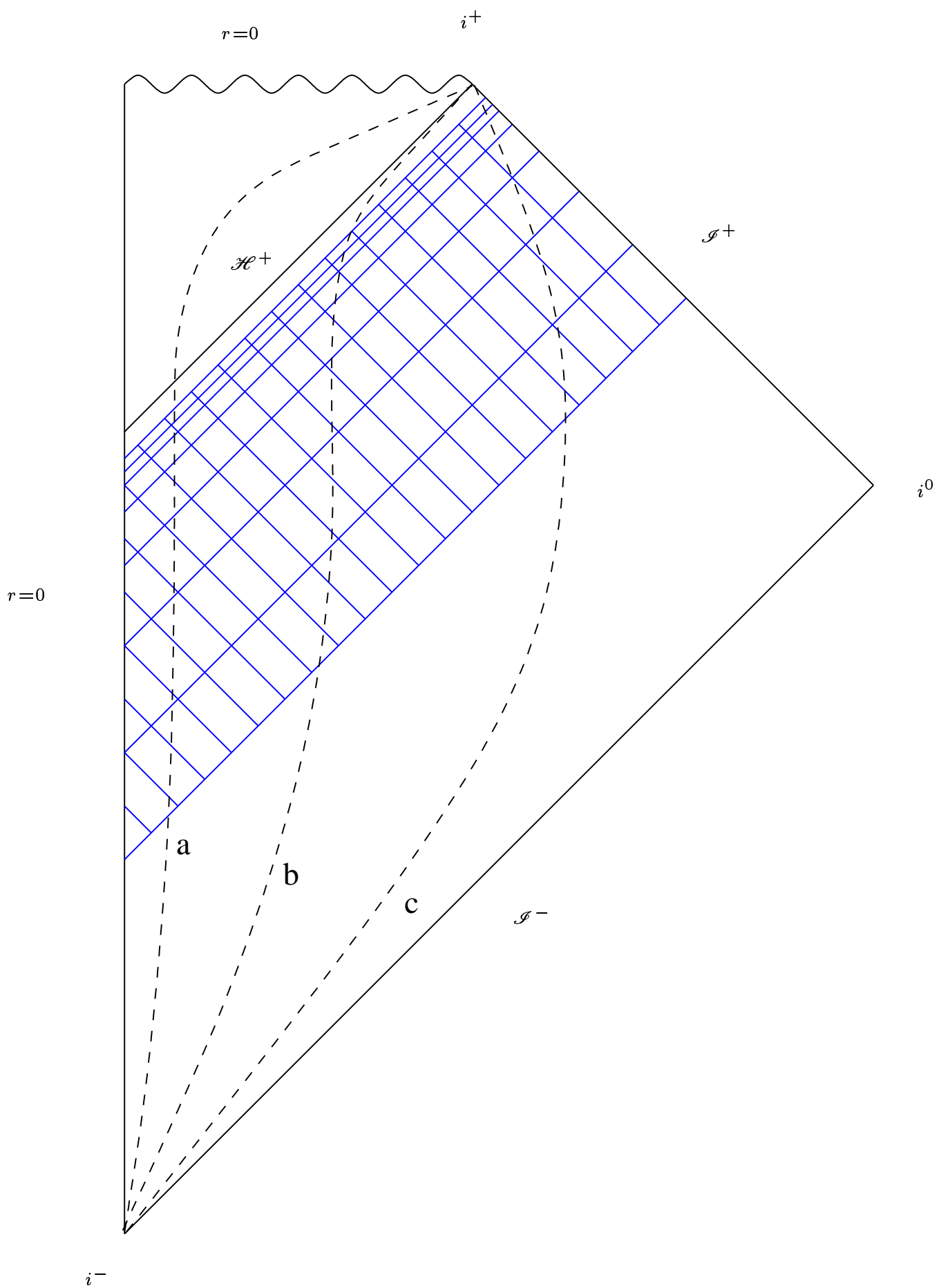


FIG. 1: A Penrose diagram of a typical collapse spacetime. Shown is our numerical null grid which extends to future null infinity \mathcal{I}^+ . The grid consists of the null slices $u = \text{const}$ and ingoing radial null geodesics $v = \text{const}$. Evolution slows down in the vicinity of the future event horizon \mathcal{H}^+ . We also indicate lines (a) $r = \text{const} < 2M_f$, (b) $r = \text{const} = 2M_f$ and (c) $r = \text{const} > 2M_f$, where M_f is the final black hole mass.

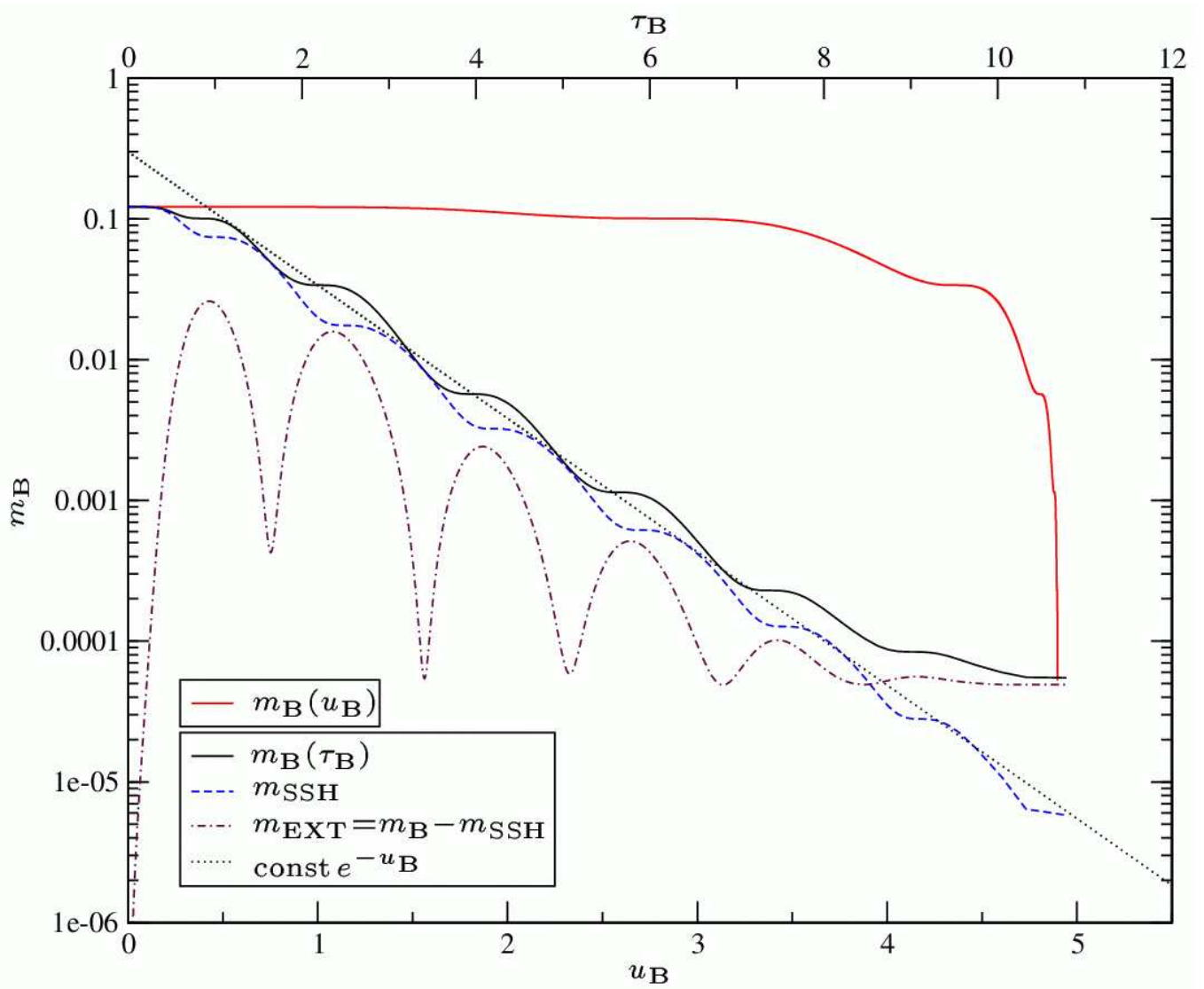


FIG. 2: This figure plots the Bondi mass m_B against both u_B and the adapted time τ_B for a barely supercritical evolution with final black hole mass $M_f \approx 5 \times 10^{-6}$. The Bondi mass m_B and the mass at the past SSH, m_{SSH} , are found to decrease exponentially in τ_B (with an overlaid τ_B -periodic oscillation with period $\Delta/2$), once the evolution has sufficiently approached the critical solution near the center of spherical symmetry. We also show m_{EXT} , the energy present outside of the SSH.

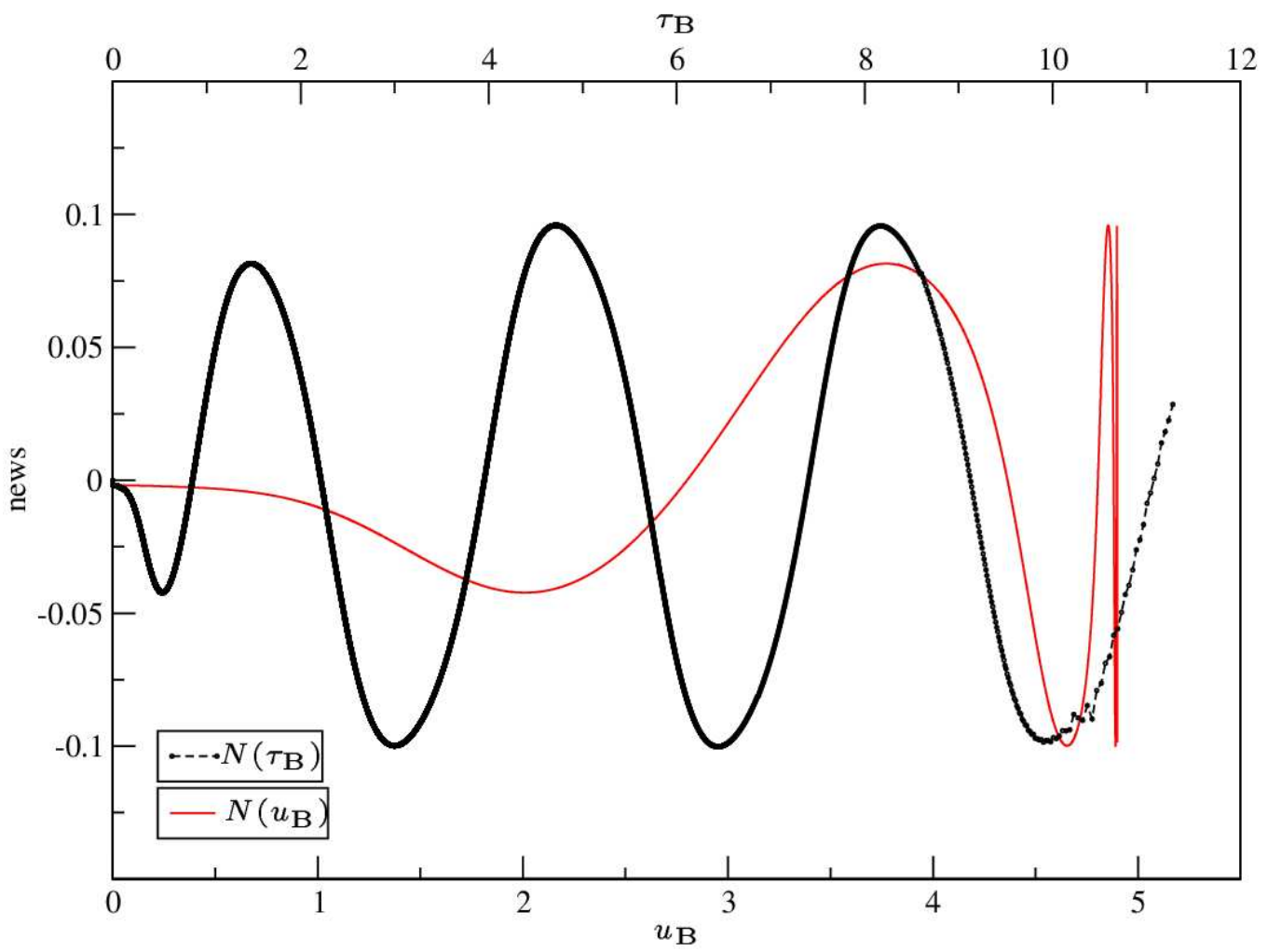


FIG. 3: We show the news function $N(u)$, first as a function of the natural time coordinate u_B of an asymptotic observer and also as a function of a suitably adapted time $\tau_B = -\ln \frac{u_B^* - u_B}{u_B}$ where $N(\tau_B)$ is periodic with period $\Delta \simeq 3.44$ after the spacetime has come close to the critical solution. Even if the constant u_B^* is not known, it can be determined by a fit to periodicity in τ_B . Thus, it is possible to observe DSS at \mathcal{S}^+ and to extract the critical exponent Δ .

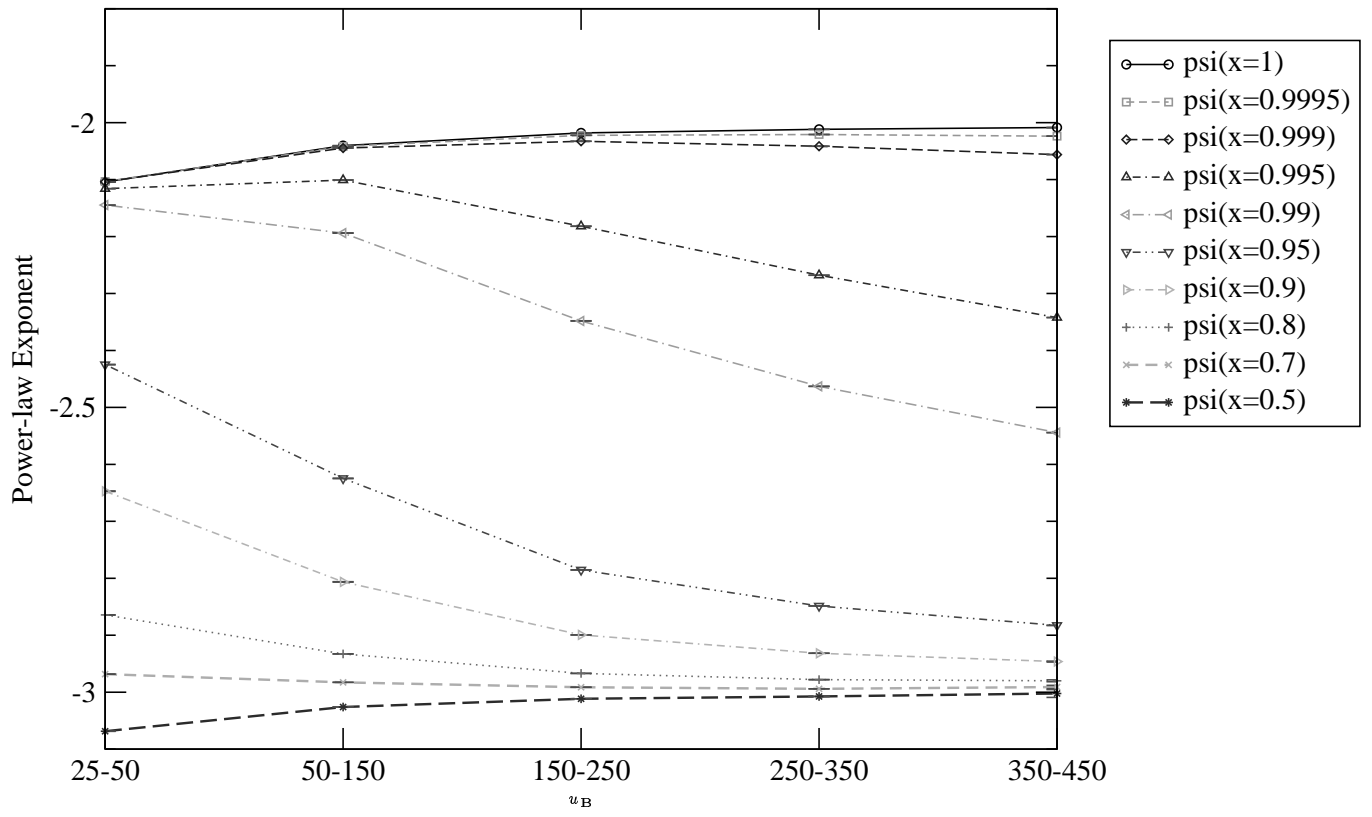


FIG. 4: This figure shows power-law exponents for a subcritical evolution and illustrates the domains of validity of the predictions of perturbation theory for the two zones: -2 near \mathcal{S}^+ and -3 near i^+ .