Recent Developments in Black Hole Critical Phenomena

Matthew W. Choptuik
CIAR Cosmology and Gravity Program
Department of Physics and Astronomy
University of British Columbia
Vancouver BC

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Outline

- Brief Review of Critical Collapse
- Spherically-symmetric Einstein-Vlasov
- Spherically-symmetric Perfect Fluid in “Dust” Limit

Review of Black Hole Critical Phenomena

- Consider parameterized families of solutions to Einstein equations, typically coupled to one or more matter fields (but vacuum case can also be considered); focus on collapse of matter/energy and black hole formation
- Family parameter, p, viewed as "control parameter" for initial data, and hence for subsequent dynamical evolution

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- Demand that family "interpolates" through the black hole threshold, i.e. that there exists a critical value, $p = p^\star$, such that

  1. $p < p^\star$: No black hole forms
  2. $p > p^\star$: Black hole forms

- Empirically (and for some models, analytically) scenarios 1. and 2. characterized by long-time, stable "end-states" of evolution, may be *only* such states

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- Solution in near-critical regime $p \sim p^\star \equiv$ black hole critical phenomena

- Use "competition" (loosely, kinetic energy vs potential energy) inherent in collapse models, and fine-tuning to dynamically evolve to *unstable* critical solution

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- Near-critical solutions characterized by scaling of dimensionful quantities (defines additional critical exponents)

Critical Phenomenology

- Although *unstable*, critical solutions tend to be *minimally* so, in the sense of having *one* unstable mode in the context of perturbation theory

- Growth factor (Lyapunov exponent), $\mathrm{Re}\lambda_1$, of unstable mode can be immediately related to exponents in scaling relations

Type I Critical Solutions

- Smallest BH has finite mass

- Model will generally have one (or more) intrinsic length scales that will set the minimum mass

- Critical solution exhibits time translational invariance

  1. Continuous: static
  2. Discrete: periodic, defines "exponent", ω

- Scaling law for, e.g., "lifetime" of near-critical configuration during dynamical evolution

$$\tau \sim \sigma \ln |p - p^\star| \qquad \sigma = [\mathrm{Re}\lambda_1]^{-1}$$

Type I Critical Solutions

- Examples (all spherically symmetric)
  - magnetic EYM ($n = 1$ Bartnik-McKinnon solution)
  - real scalar field (unstable oscillons, Brady et al)
  - complex scalar field (unstable mini-boson stars, Hawley, Lai)
  - perfect fluid (neutron star models on unstable branch, Noble)

Type II Critical Solutions

- No minimum BH mass, arbitrarily small BHs possible
- Critical solution exhibits scale invariance
  1. Continuous: continuous self-similarity (CSS)
  2. Discrete: discrete self-similarity (DSS), defines "echoing exponent", Δ
- Scaling law for, e.g., BH masses from super-critical evolutions:

$$\ln M_{\mathrm{BH}} \sim \gamma \ln |p - p^{\star}| \qquad \gamma = [\mathrm{Re}\lambda_1]^{-1}$$

Type II Critical Solutions

- Examples (spherically symmetric)
  - massless scalar field: $\Delta \approx 3.44$, $\gamma \approx 0.37$
  - magnetic EYM: $\Delta \approx 0.74$, $\gamma \approx 0.20$
  - non-linear sigma models (Choptuik *et al*, Husa *et al*)
  - perfect fluid (Evans & Coleman, Neilsen, Noble)
- Examples (axisymmetric)
  - vacuum gravitational waves (Abraham & Evans)
  - massless scalar field with angular momentum (Pretorius *et al*)

Near Critical Evolution in Einstein-Vlasov Collapse

- ANIMATION of marginally sub-critical evolution
- ANIMATION of marginally super-critical evolution