

In the situation of two vortices in a thin slab, the free energy is:

$$F = 2[F_1 + \sum_{n=-\infty}^{\infty} U(nw) + \sum_{n=-\infty}^{\infty} U(\sqrt{x^2 + (nw)^2})] \quad (1)$$

we map this to a potential function  $V(x)$  and ignore the first two terms because they do not depend on  $x$ . We then have

$$V(x) = 2 \sum_{n=-\infty}^{\infty} U(\sqrt{x^2 + (nw)^2}) \quad (2)$$

where  $U$  is defined by

$$U(r) = \frac{\phi_0^2}{8\pi^2\lambda^2} K_0\left(\frac{r}{\lambda}\right) \quad (3)$$

and  $K_0$  is the  $0^{th}$  order Bessel function. Use this potential in the Schrodinger equation with

$$H\psi = E\psi \quad (4)$$

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \quad (5)$$

We expect  $\psi$  asymptotically to be of the form:

$$\psi = \sin(k|x| - \delta(k)) \quad (6)$$

and  $\delta(k)$  to be

$$\delta(k) = ak \quad (7)$$

once this is done, the goal is to determine the value of  $a$ , because  $a$  is involved in an equation for another parameter  $g$ , which is what we are really interested in:

$$g \approx 1 - 2an_0 \quad (8)$$

where  $n_0$  is the density of vortices.