Numerical Investigation of the Schrödinger-Poisson System

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Outline

- <u>Schrödinger-Poisson (SP) System</u>
- <u>Goals</u>
  - Gravitational collapse, Singularity formation
- <u>Numerical Implementation</u>
- <u>Results: SP System</u>
- <u>Introduce Non-linearities</u>
- <u>Results: Modified SP System</u>

Matter Model

Complex massive scalar field in spherical symmetry.

Schrodinger-Poisson System

Newtonian limit of Einstein-Klein-Gordon equation

$$\begin{cases} i\dfrac{\partial \Psi}{\partial t} = -\dfrac{1}{2}\nabla^2\Psi + gV\Psi \\ \nabla^2 V = |\Psi|^2 \end{cases}$$

$$c = G = \hbar = 1$$

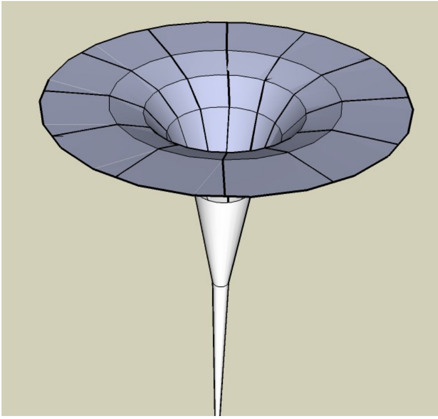
Partially motivated by work on critical collapse of Newtonian isothermal gas

Looking For ...

<u>"Blow-up Solutions"</u>

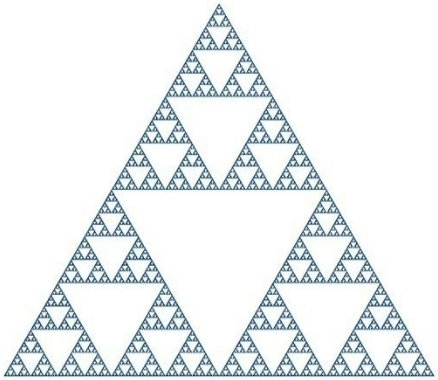
-Challenging to do numerically.

-Provide clues to the nature of singularity formation.

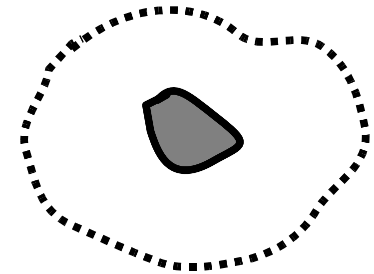


<u>Self Similarity</u>

-Common feature of gravitational collapse in spherical symmetry.



Singularities

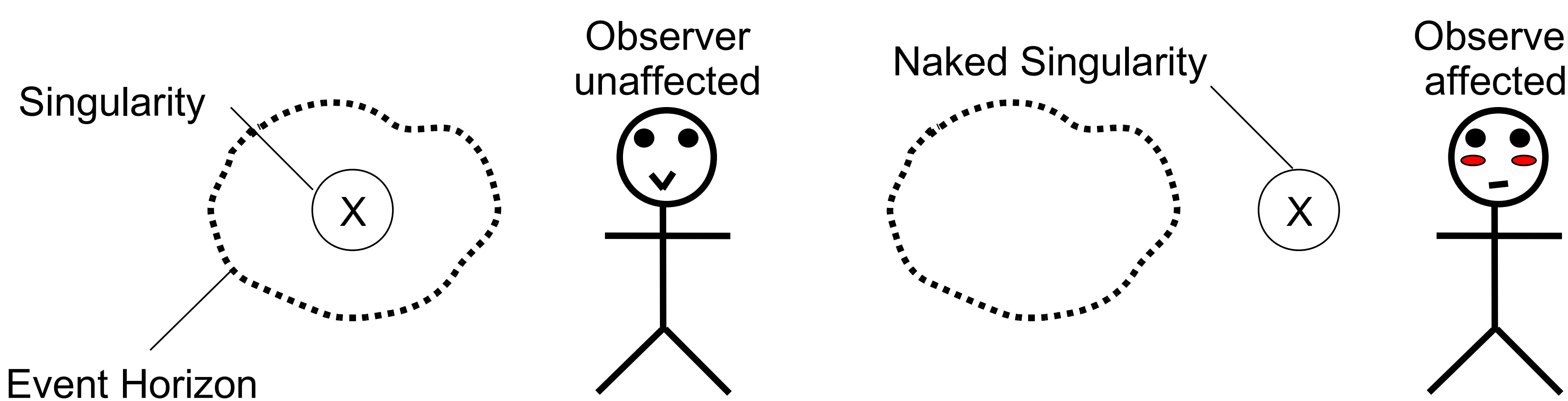


Penrose-Hawking Singularity Theorems (1960's)

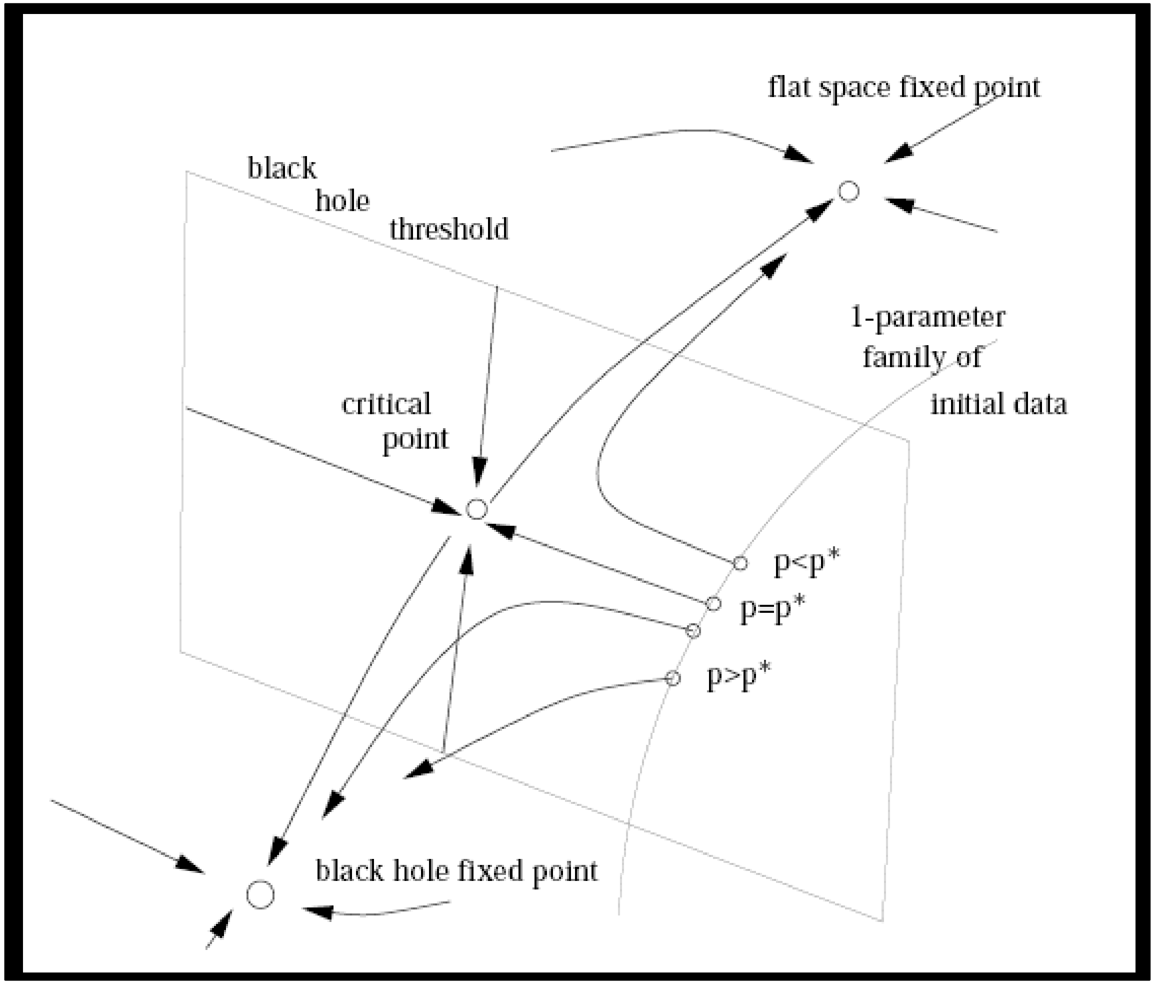
Guarantee singularity formation of sufficiently dense mass and energy configurations.

Sufficiently weak gravitating systems **never** become singular

Naked Singularities



Phase Diagram



Gundlach, 2002

Resources

Code:

RNPL (Rapid Numerical Prototyping Language)

-Simplifies input of information necessary to solve PDE's

Manual updates written in FORTRAN 77.

Visualization:

XVS: Time-dependent PDE's

Discretize Equations of Motion

<u>*Crank-Nicholson:*</u> $\mathcal{O}(h^2)$

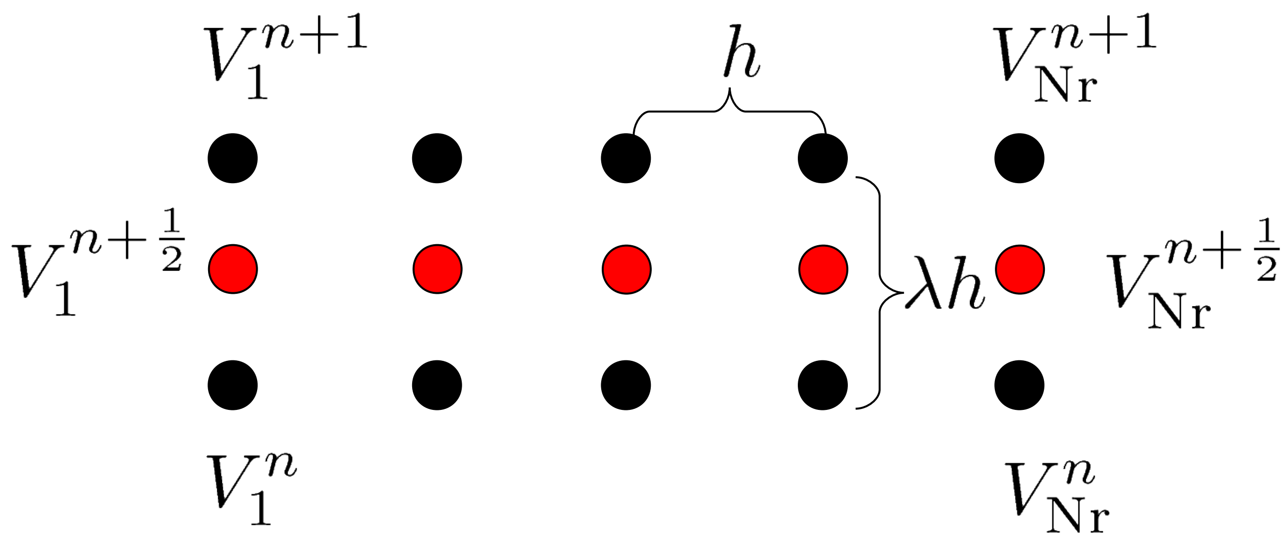
$$\left(\frac{\partial V}{\partial t}\right)_j^{n+\frac{1}{2}} \approx \frac{V_j^{n+1} - V_j^n}{\lambda h}$$

<u>*Leap-Frog Operator:*</u> $\mathcal{O}(h^2)$

$$\left(\frac{\partial V}{\partial r}\right)_j^{n} \approx \frac{V_{j+1}^{n} - V_{j-1}^n}{2h}$$

In Spherical Symmetry:

$$\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right)$$



Boundary Conditions

Inner

$$\frac{dV}{dr}\Big|_{r=0} = 0$$

$$\frac{d\Psi}{dr}\Big|_{r=0} = 0$$

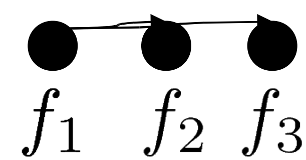
Outer

$$V\big|_{r=1} = 0$$

$$\Psi\big|_{r=1} = 0$$

Forward-difference leap frog operator:

$$\frac{-\frac{3}{2}f_1^n + 2f_2^n - \frac{1}{2}f_3^n}{h} = (f_r)_1^n + \mathcal{O}(h^2)$$



$$\begin{cases} V_1^n = -\dfrac{1}{3}V_3^n + \dfrac{4}{3}V_2^n \\ \\ V_{\mathrm{Nr}}^n = 0 \end{cases}$$

Tridiagonal Form

$$\begin{cases} d_-\Psi_{j-1}^{n+1} + d_0\Psi_j^{n+1} + d_+\Psi_{j+1}^{n+1} = F_j^{n+1} \\ c_-V_{j-1}^{n+1} + c_0V_j^{n+1} + c_+V_{j+1}^{n+1} = \left(\Psi_j\Psi_j^*\right)^{n+1} \end{cases}$$

Back and forth until converge

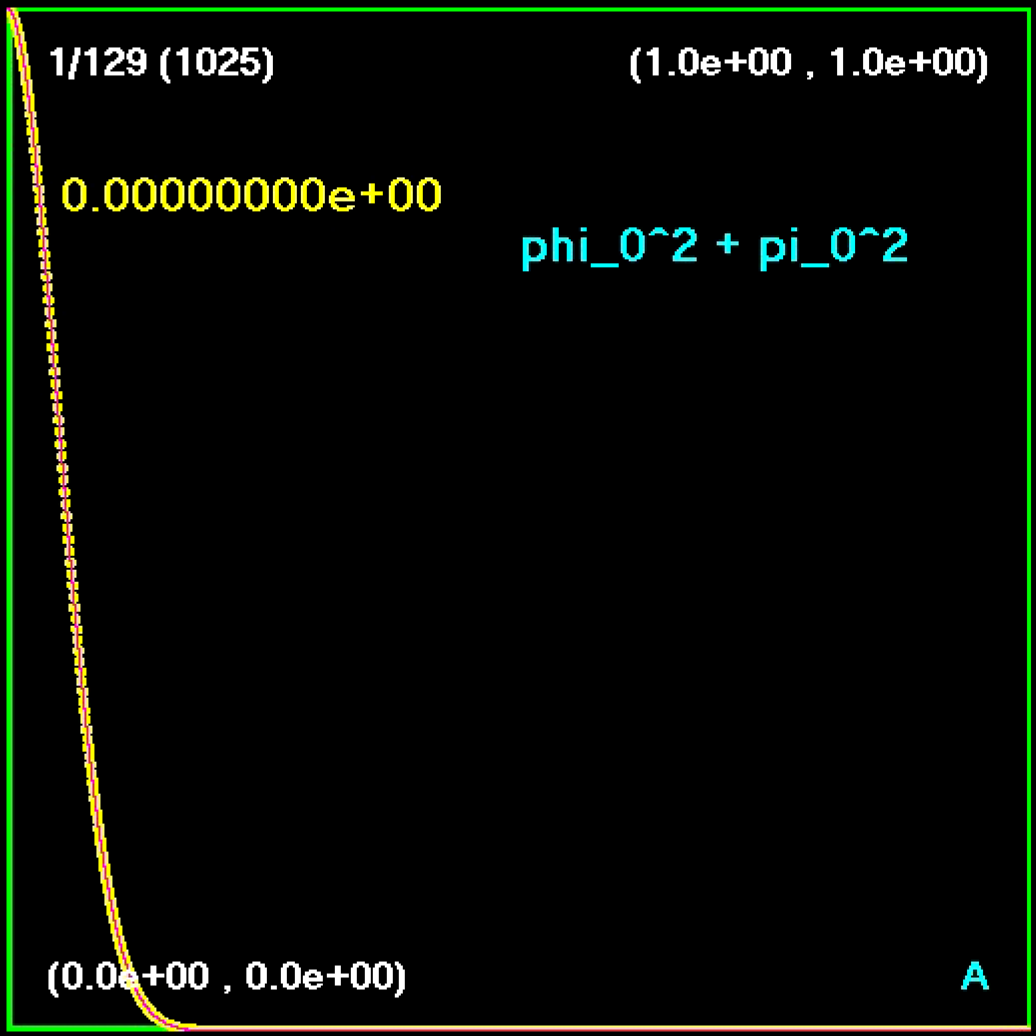
$$\begin{pmatrix} d_0 & d_+ & 0 & 0 \\ d_- & d_0 & d_+ & 0 \\ 0 & d_- & d_0 & d_+ \\ 0 & 0 & d_- & d_0 \end{pmatrix} \begin{pmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \end{pmatrix} = \begin{pmatrix} \Psi_2 \\ \Psi_3 \\ \Psi_4 \\ \Psi_5 \end{pmatrix}$$

Invert using routines in LAPACK (Linear Algebra PACKage)

SP System

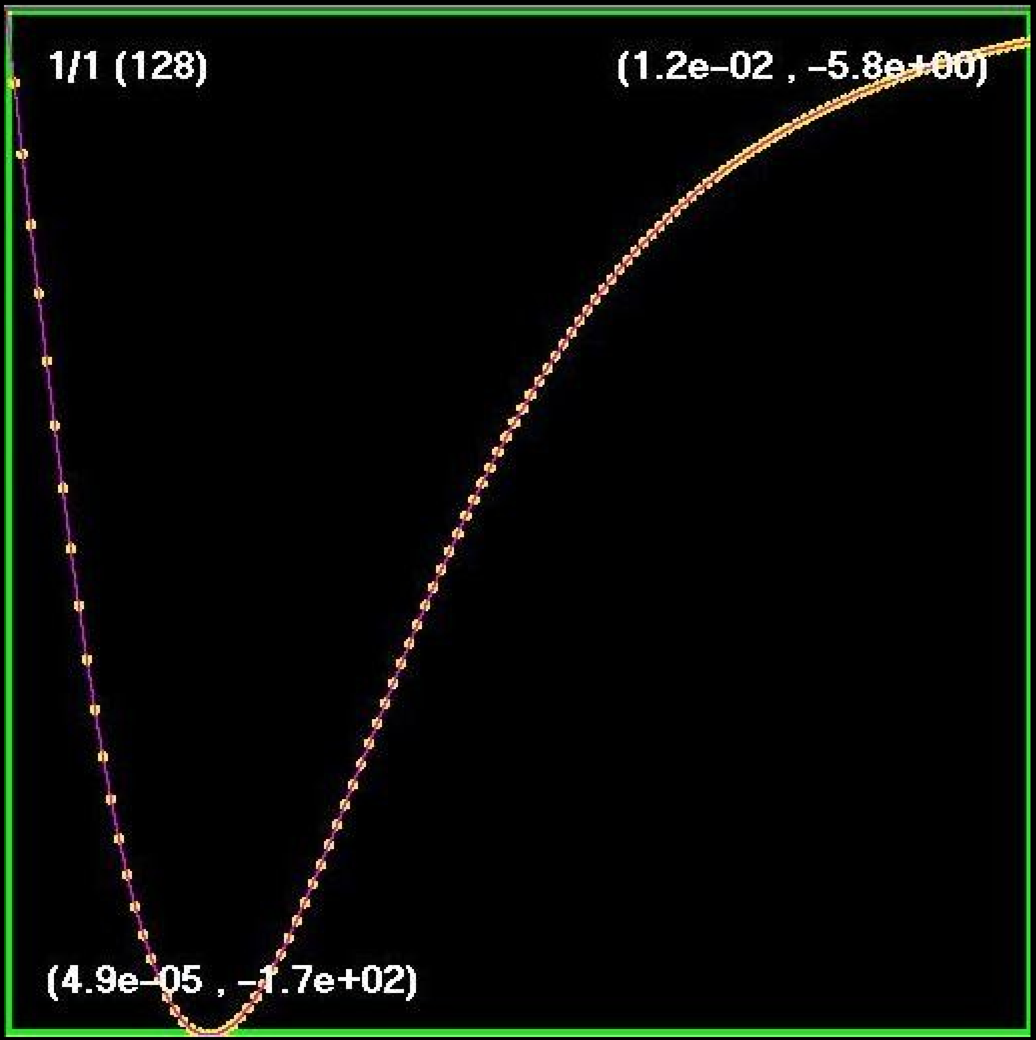
g = 1
A = 1

$|\Psi|^2$



r

$\frac{\partial|\Psi|^2}{\partial t}|_{r=0}$



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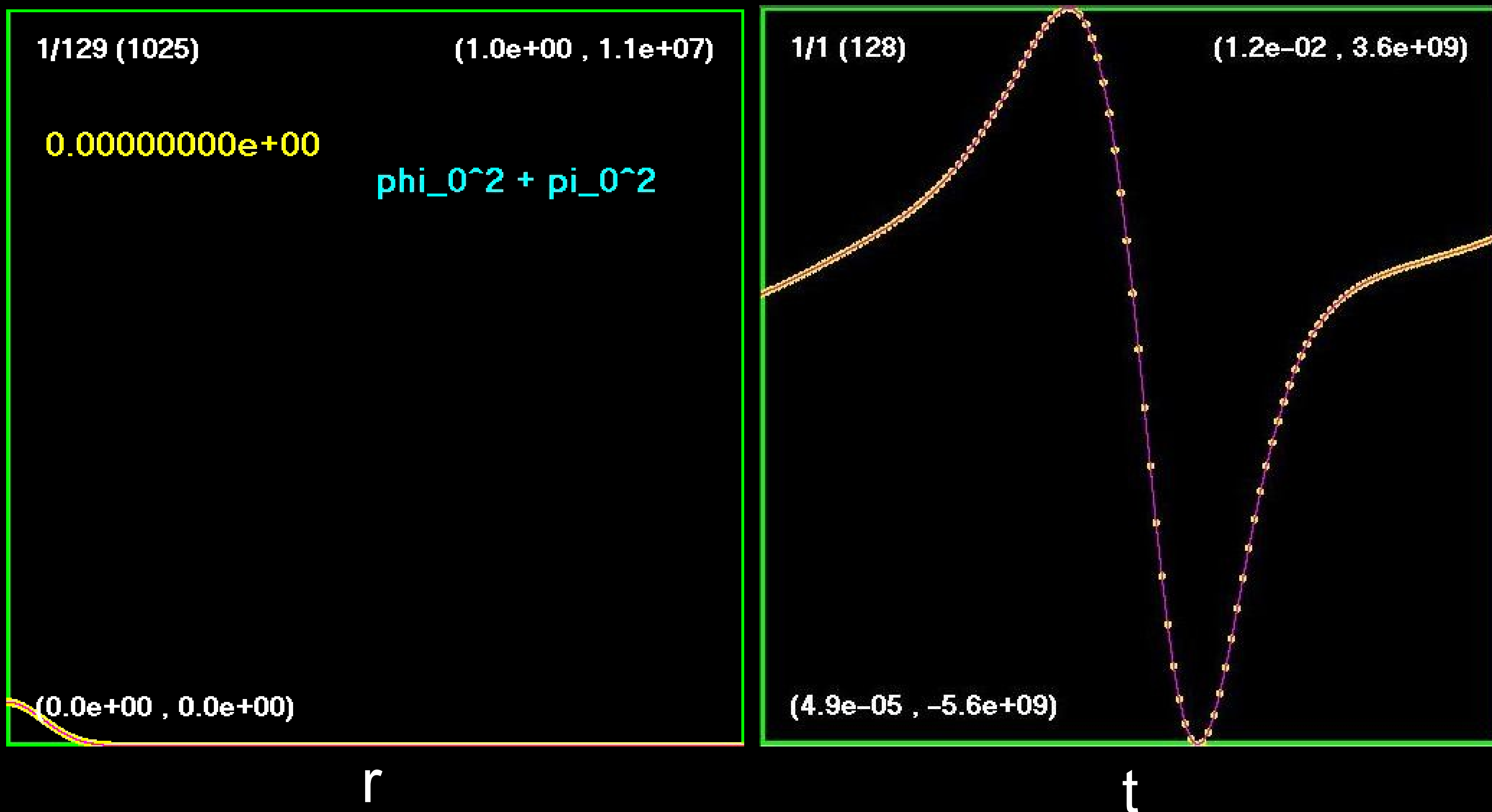
SP System

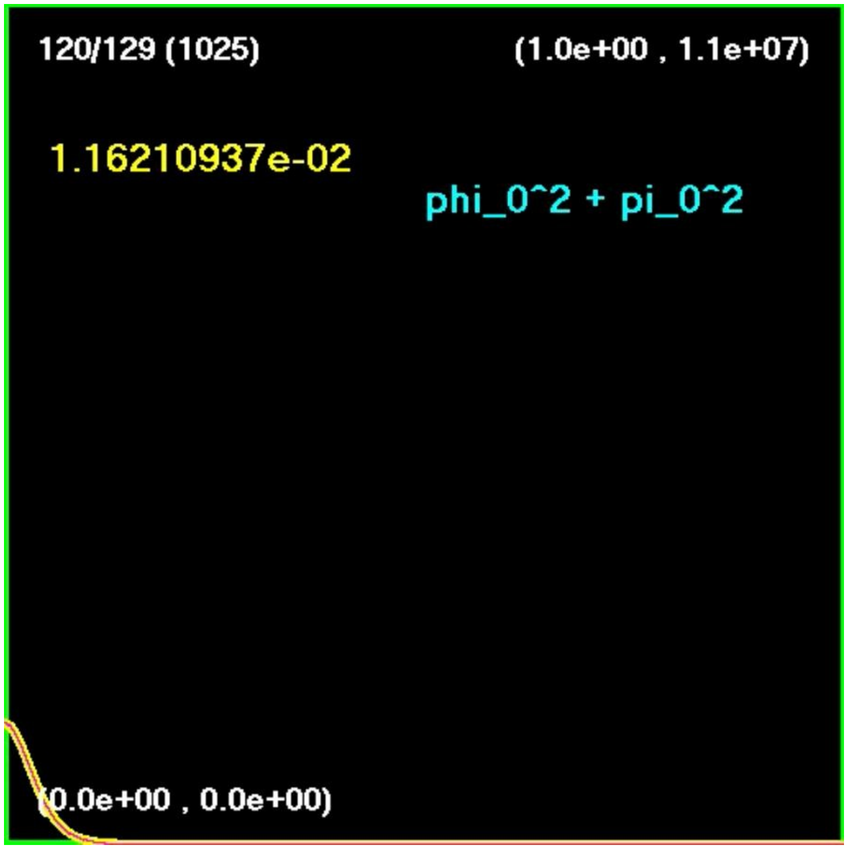
g = 1
A = 800

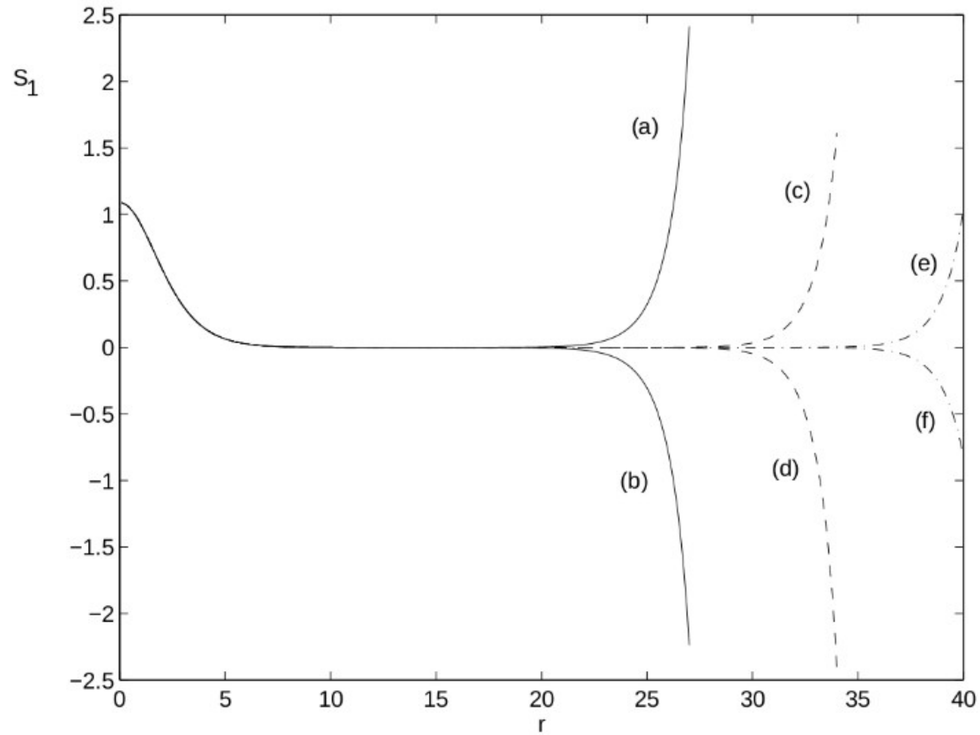
$|\Psi|^2$ $\qquad$ $\frac{\partial |\Psi|^2}{\partial t}\Big|_{r=0}$



Comparison with Previous Work

Infinite set of static solutions called “Newtonian Boson Stars”.



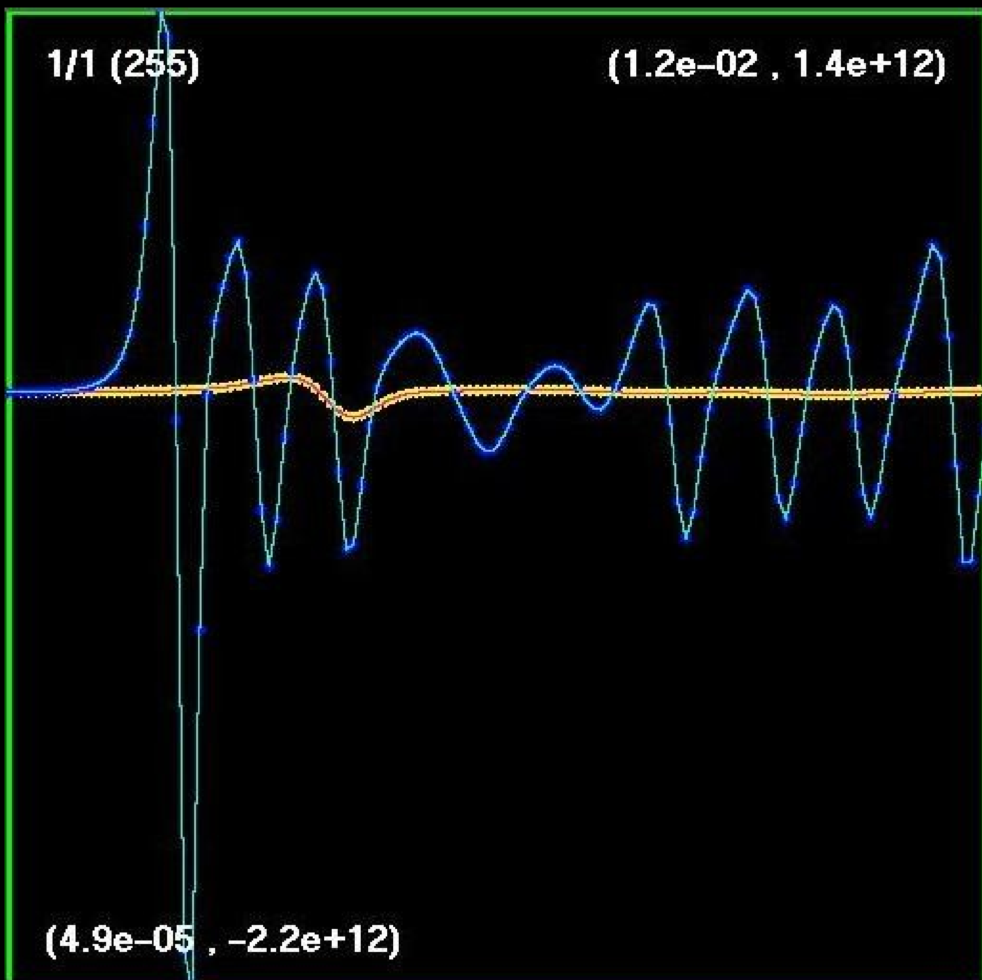


Moroz, Penrose, Tod, 1998

Search for Critical Behaviour

$$\frac{\partial|\Psi|^2}{\partial t}\Big|_{r=0}$$

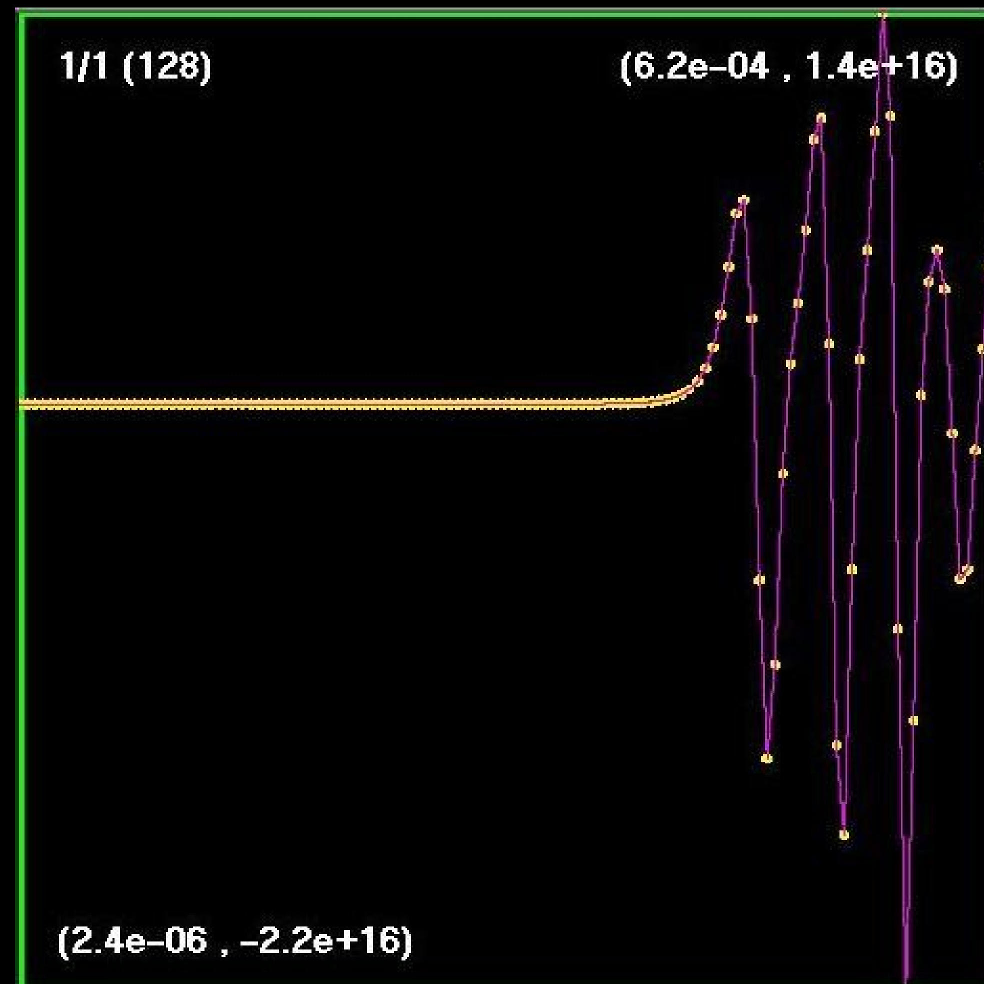
A = 1000
A = 1500



t

$$\frac{\partial|\Psi|^2}{\partial t}\Big|_{r=0}$$

A = 5000



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Change Matter Model to Induce Blow-up

Modified Schrödinger Poisson System

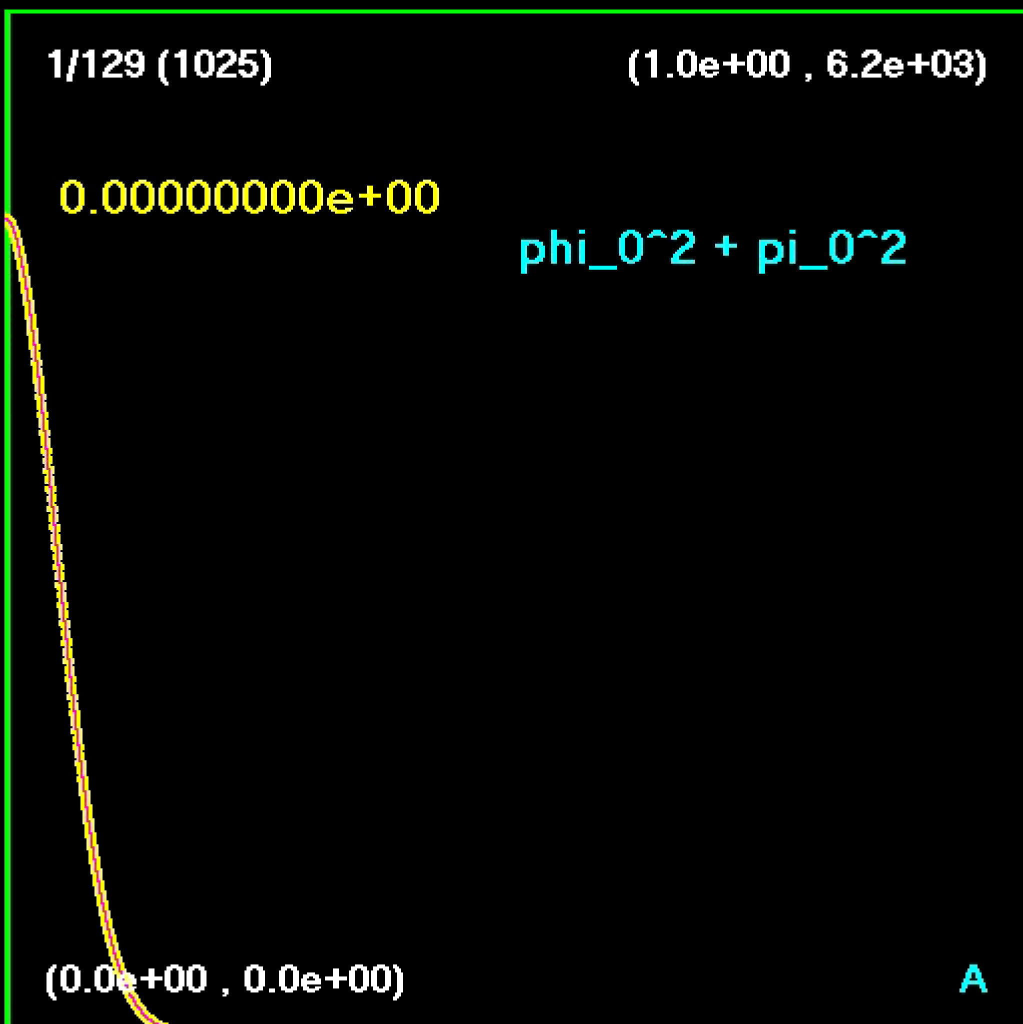
$$\begin{cases} i\dfrac{\partial \Psi}{\partial t} = -\dfrac{1}{2}\nabla^2\Psi + gV\Psi \\ \nabla^2 V = |\Psi|^{\epsilon} \end{cases}$$

So far have done investigations where $\epsilon = 3$

Modified SP System

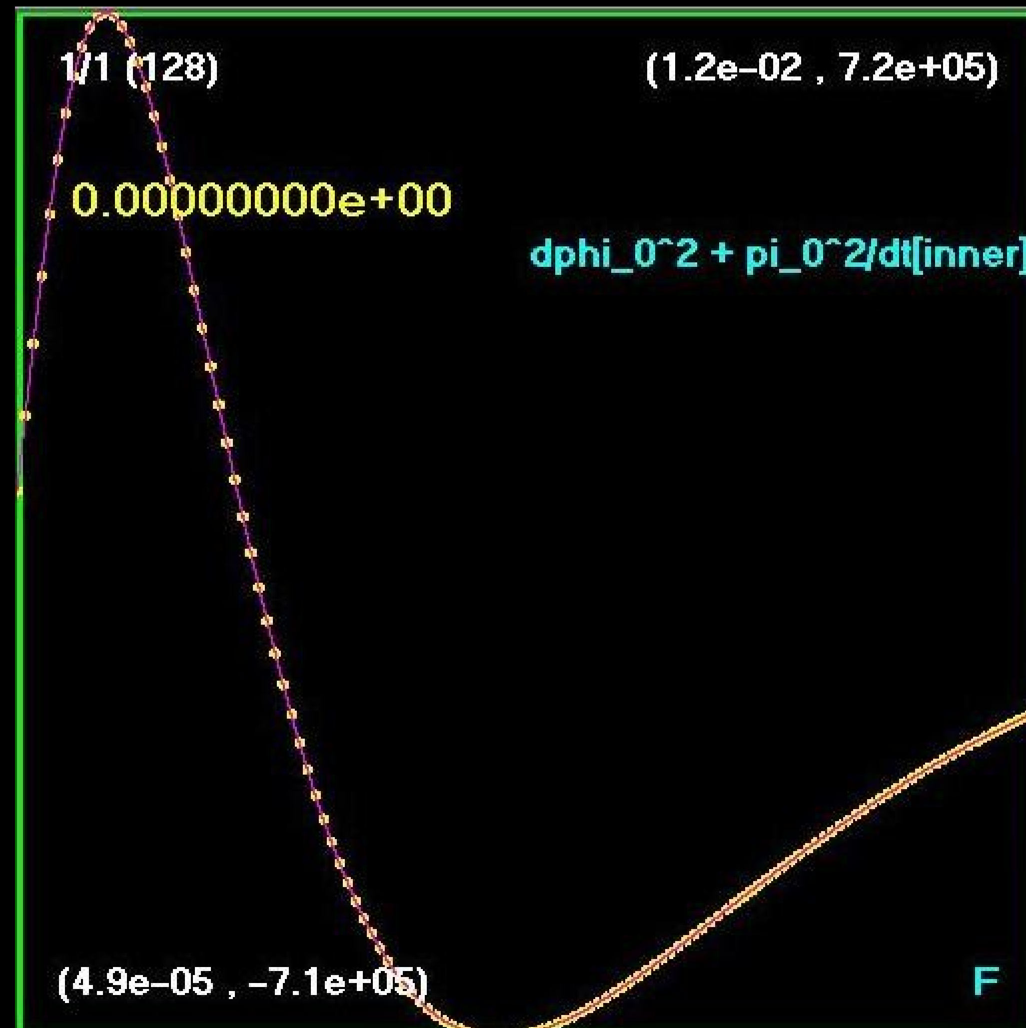
g = 1
A = 70

$|\Psi|^2$



r

$\frac{\partial|\Psi|^2}{\partial t}|_{r=0}$



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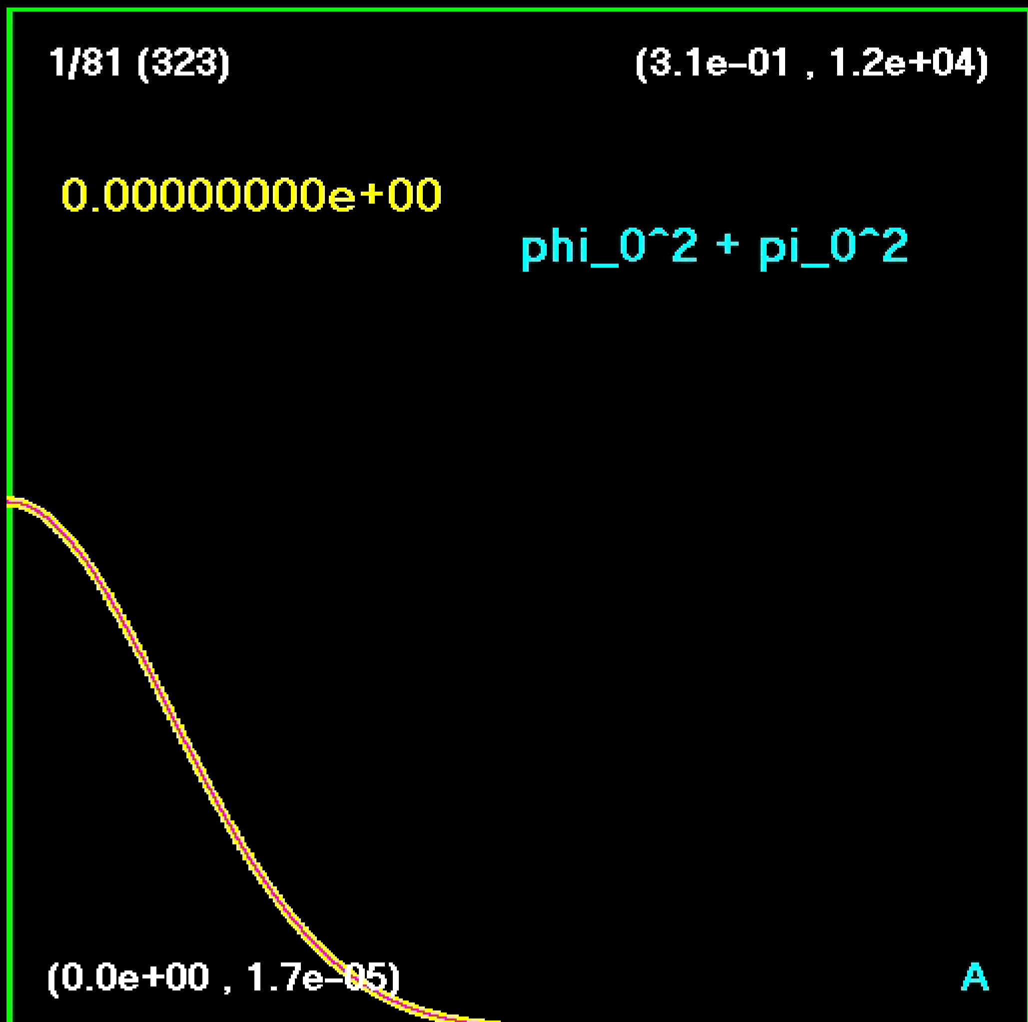
Modified SP System

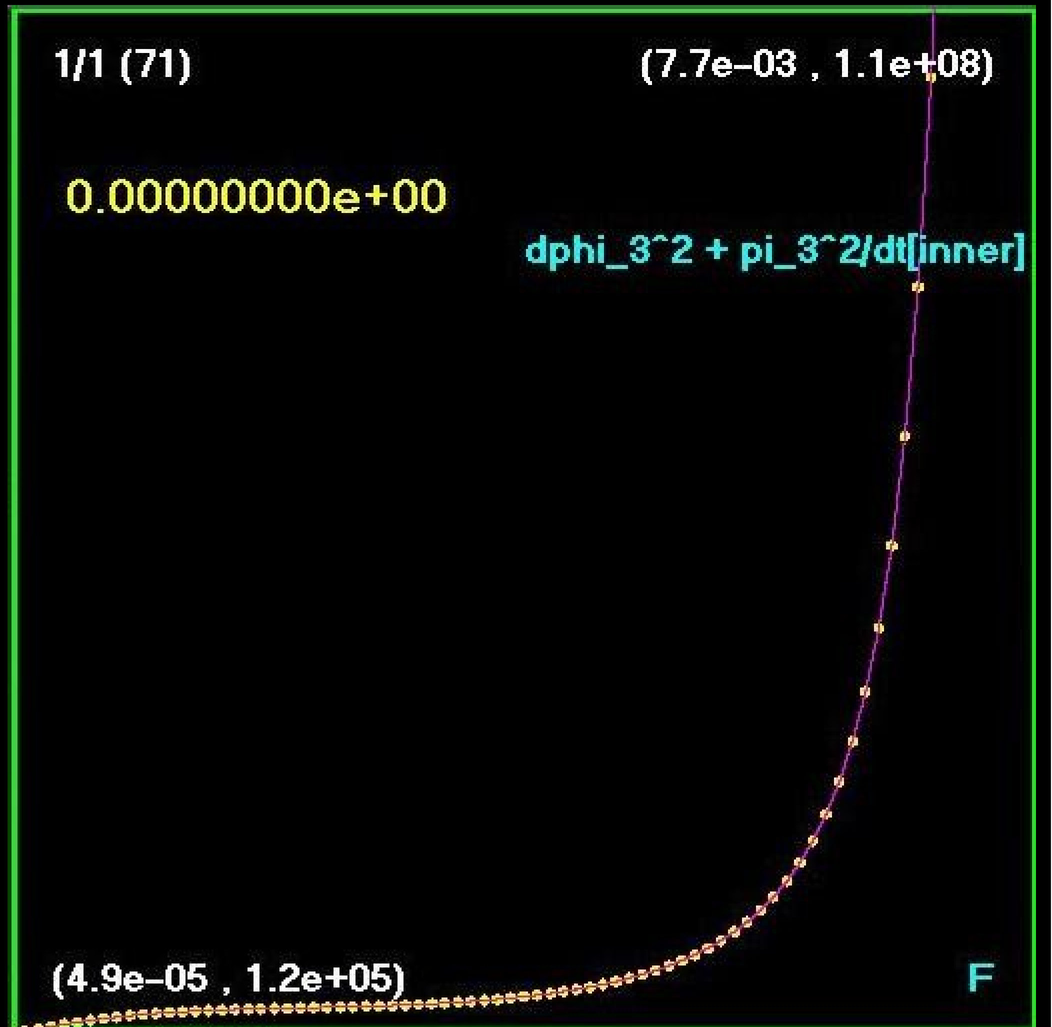
g = 1
A = 80

$$|\Psi|^2$$



r

$$\frac{\partial |\Psi|^2}{\partial t}\Big|_{r=0}$$



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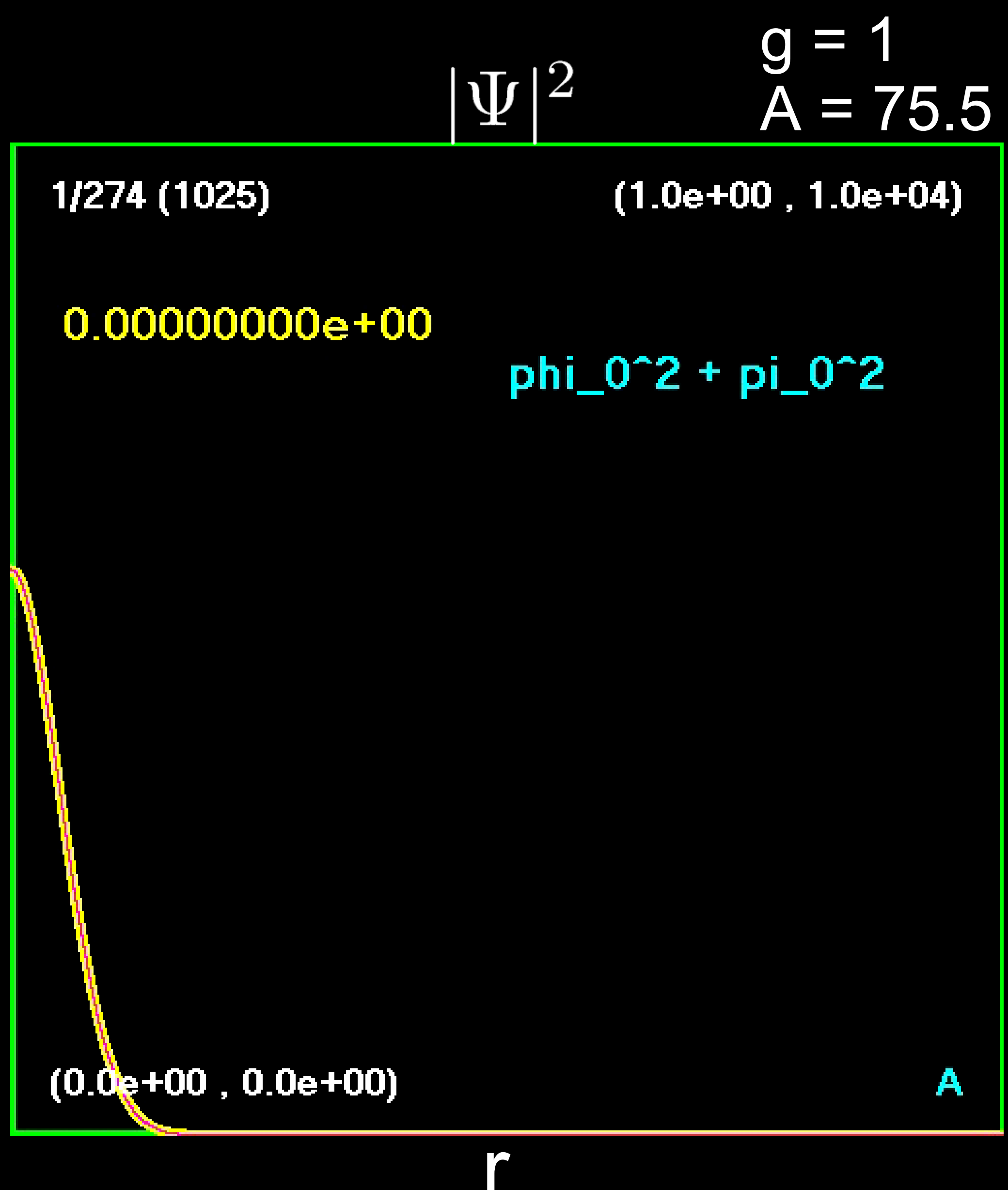
Critical Solution?

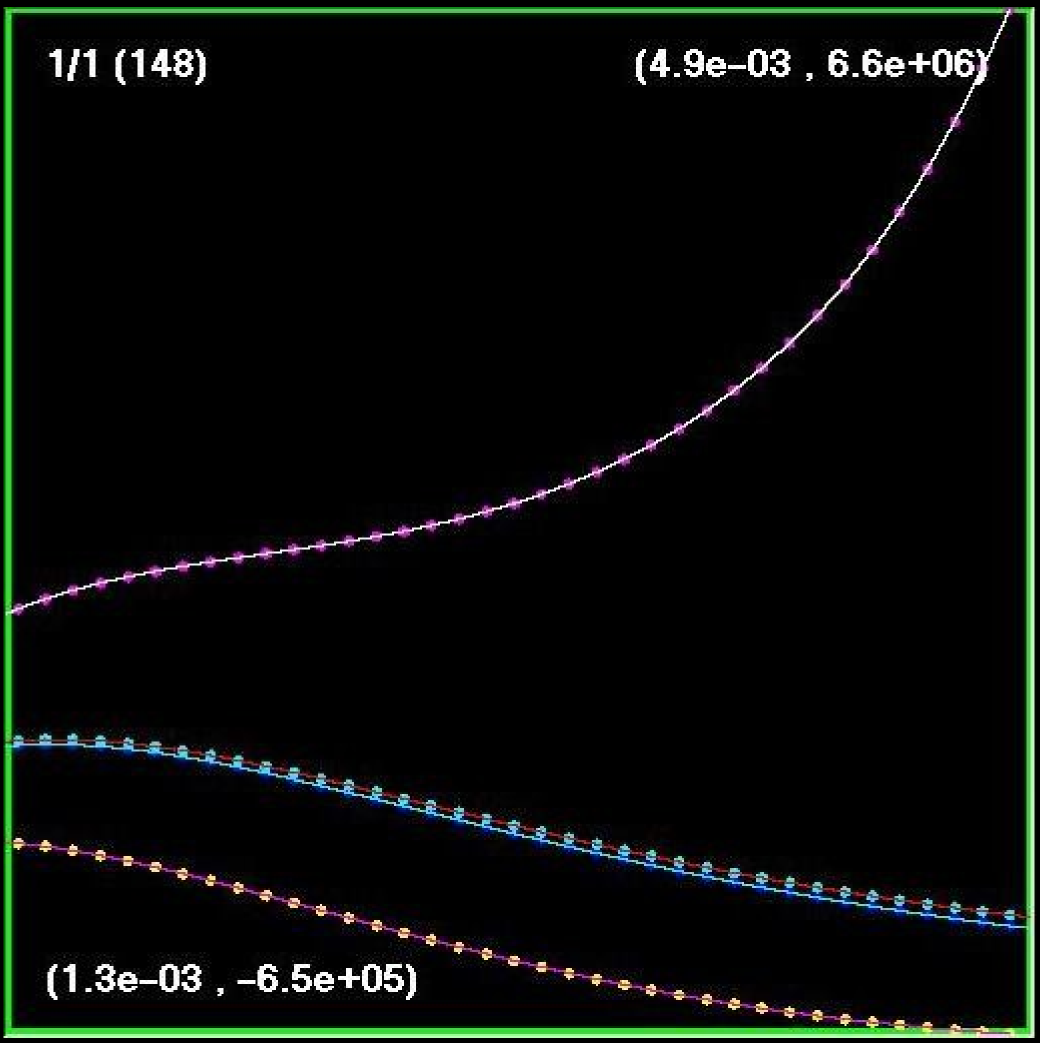


Comparison of Growth Rates

A = 80
75.5
75.3
70

$$\frac{\partial|\Psi|^2}{\partial t}\Big|_{r=0}$$

$$\frac{\partial|\Psi|^2}{\partial t}\Big|_{r=0}$$



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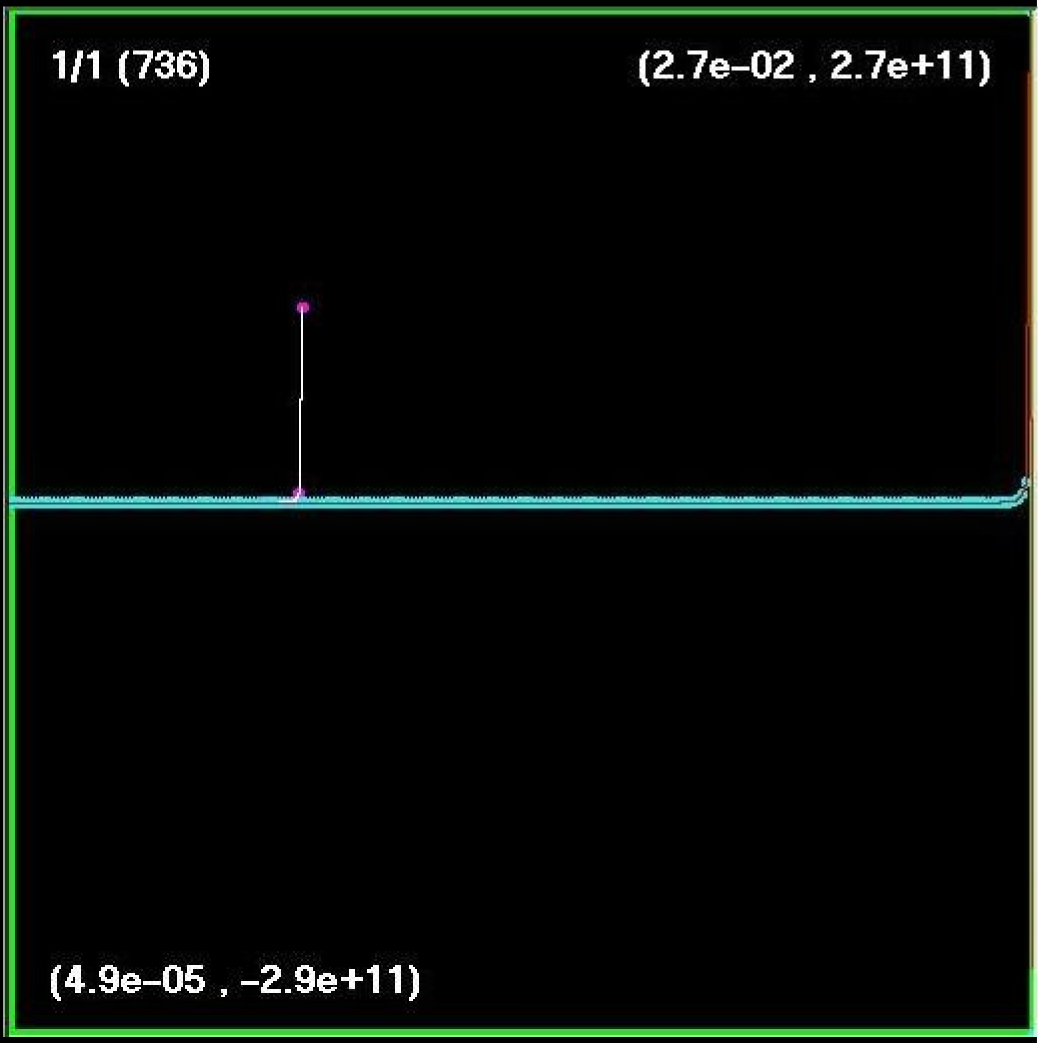

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Summary

- Schrödinger-Poisson (SP) System was solved numerically using a Crank-Nicolson finite-difference approximation scheme

- Varying a 1-parameter family of initial data, no evidence of blow-up was found.

- Introduced a non-linearity into SP system, through the gravitational coupling.

- Blow-up solutions were found for sufficiently large non-linearity.

- Evidence for a critical solution at the threshold between dispersive and blow-up solutions.

Future Outlook

- Find threshold on epsilon for which blow-up solutions do not occur.

- Construct the exact form of the critical solution.

- Improve treatment at outer boundary to allow for longer integration.

- Implement mesh refinement techniques.

- Test stability of critical solutions and blow-up outside of spherical symmetry.

- Try to solve equations in cylindrical symmetry or other.